Analytic Modelling of the Shear Layer in a Laser-Driven Water Microjet with a Magnetohydrodynamic Shear-flow Stabilized Z-pinch Equilibrium

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We investigate whether an analytic shear-flow-stabilized Z-pinch equilibrium can describe the shear layer observed in laser-driven water-microjet experiments from measurements of radiant intensity. Using experimentally constrained expansion velocities to set the boundary flow, we construct families of analytic equilibrium solutions using the experimental state of the system, and present branches satisfying reality, subluminality, and localization criteria. These analytic branches obtained from an ideal magnetohydrodynamic model satisfy $R^2 > 0.90$, and number in the hundreds for a sweep dimension of $NV = 25$.

Introduction.—Laser-driven liquid microjets provide a platform for studying transient, strongly driven plasma and fluid interfaces. Recent BELLA microjet measurements [1] provide a spatially resolved intensity profile against which an analytic, magnetohydrodynamic shear-flow stabilized Z-pinch[2][3] equilibrium model can be tested. The data being studied is shown in panel (b.) of Figure (1).

Equilibrium model.—We study a magnetohydrodynamic shear-flow stabilized equilibrium obtained from exchanging the non-relativistic Bennett nonlinearity[4][5]

$$n(r) = \frac{n_0}{(1+\xi^2 r^2)^2} \tag{1}$$

$$\xi^2 = \frac{C_B n_0}{T} \tag{2}$$

$$C_B = \frac{\mu_0 e^2 u_{z,0}^2}{16 k_B} \tag{3}$$

from density to flow velocity. In the presence of an enhanced thermodynamic gradient,

$$T(r) = \frac{T_p}{r_p^3} r^3 \tag{4}$$

the flow velocity is arrived at,

$$u_z(r) = u_{z0} \frac{r^2}{(r+C)^2}, \tag{5}$$

when the drift flow is treated as the center-of-mass flow so that this profile can be used both to provide the stabilizing flow shear, and the self-generated azimuthal magnetic field. Treating the plasma flows in this manner holds in a few physical cases such as when one species carries the majority of the momentum, e.g., non-relativistic hydrogen ions in a singly-ionized hydrogen-electron plasma.

C is a characteristic radial scale for the shear layer and u_{z0} sets the velocity amplitude. The corresponding shear magnitude is

$$\left| \frac{du_z}{dr} \right| = \left| \frac{2 u_{z0} C r}{(r+C)^3} \right|. \tag{6}$$

The shear is maximal at $r = C/2$ for $C > 0$,

$$C = \frac{\mu_0 e^2}{16 k_B} \frac{n_0 u_{z0}^2 r_p^3}{T_p} \tag{7}$$

and the profile satisfies ideal magnetohydrodynamics because of its stationary, uniform, axisymmetric Z-pinch character provided the force balance integrates,

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -J_z B_\theta \tag{8}$$

which it does for the magnetohydrodynamic system described here. To account for a non-zero core flow in the models presented here we solve the profile,

$$u_z(r) = u_0 + u_{z0} \frac{r^2}{(r+C)^2} \tag{9}$$

The system is closed by an isotropic heat flux closure, rather than the ideal gas law, as shear-flow stabilized Z-pinches do not behave ideally[6],

$$\vec{q} = -\kappa_\perp \nabla T \tag{10}$$

The shear-flow stabilization property of this plasma closure is strongly-satisfied arbitrarily towards the axis both in the ideal,

$$\frac{\mathrm{d}u_z}{\mathrm{d}r} > 0.1 \frac{B_\theta}{\sqrt{\rho \mu_0}} \tag{11}$$

and non-ideal,

$$\frac{\mathrm{d}u_z}{\mathrm{d}r} > \frac{L}{2\pi r_p} \left| \frac{\mathrm{d}\ln(p)}{\mathrm{d}\ln(r)} \right| \tag{12}$$

cases due to the transcendental structure of the magnetic field, and plasma pressure.

Experimental constraints and boundary conditions.

A number density of $n_0 = 10^{23}\ [m^{-3}]$ is taken alongside an ion temperature of $T_p = 5\ [eV]$. The edge velocity is calculated as the center-of-mass speed obtained from the expansion velocities,

$$u(r_p) = u_{edge} \tag{13}$$

$$= \frac{m_H}{m_H + m_{OX}} u_H + \frac{m_{OX}}{m_H + m_{OX}} u_{OX} \tag{14}$$

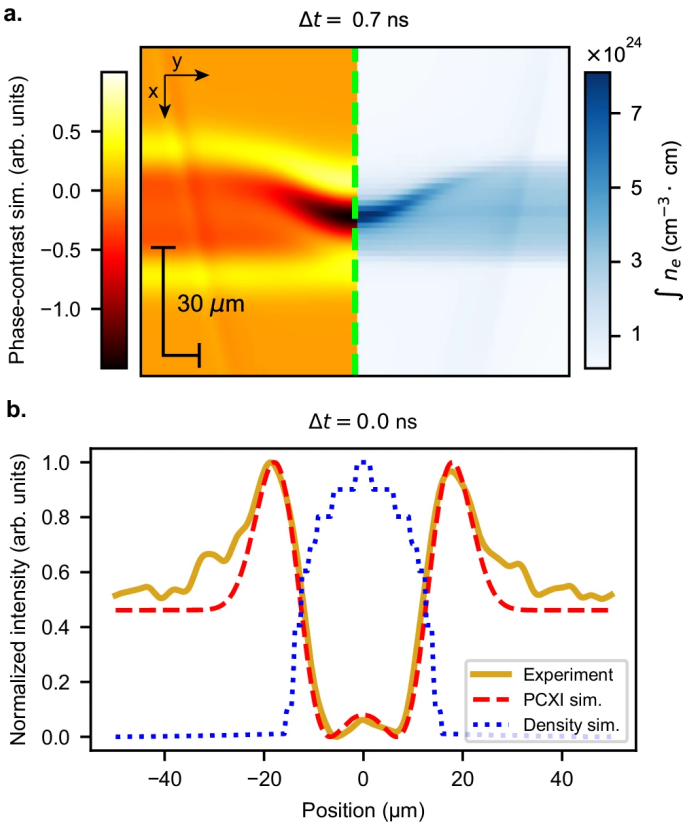


FIG. 1. The experimental data being digitized and studied in this work are shown in (b.) of this figure which represents chord-resolved intensity of the laser-driven shock in the water microjet. Interestingly, the simulated density of the system results in a self-similar structure that is approximately uniform both globally and locally.

The uncertainty in these measurements provides the basis for sweeping over and constructing solutions at each data point. The pinch radius is obtained by fixing a point as the base of the pinch, for example, where the intensity is the lowest, and then calculating the distance between that and the point chosen as the edge. Multiple candidates for the edge were considered based on being within a threshold of the maximum intensity.

The intensity of the system is translated into a current density through a linear discharge model,

$$I = \alpha J \tag{15}$$

Solving the shear-flow stabilized Z-pinch model presented here is based on finding the roots to a quartic algebraic system. These roots are not necessarily completely real or sub-luminal. Therefore, only solutions produced with real, and subluminal roots are presented in this work.

Results A threshold of 0.90 the maximum intensity was chosen to select the edge points for the modelling. Figures (2) - (5) show a representative sample of the results when only real, sub-luminal solutions are considered. The edge points close to the shear layer result in accurate models, whereas when the edge point drifts away from the shear layer the accuracy of the solutions also drifts.

Each of the figures displaying models of the shear layer represents hundreds of solutions calculated over the experimental velocity sweep with $NV = 25$ and uncertainty $u_{H,unc.} = 39\ [km/s]$, and $u_{OX,unc.} = 21\ [km/s]$ providing bounds for the sweep. Statistics for the sweeps over $NV^2 = 625$ total edge velocities are shown in Table (I).

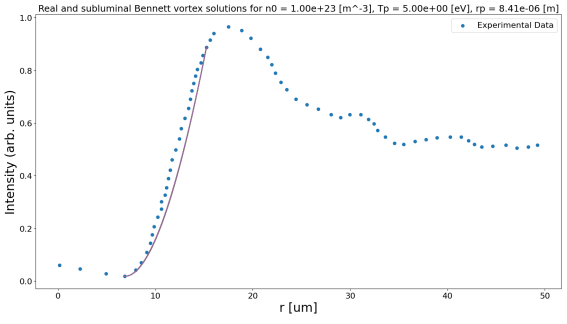


FIG. 2. Shear-flow stabilized solutions for the shear layer in a laser-driven shock of a water microjet using the closest edge point as obtained from the threshold. Every single of the $NV^2 = 625$ solutions shown here has $R^2 > 0.90$.

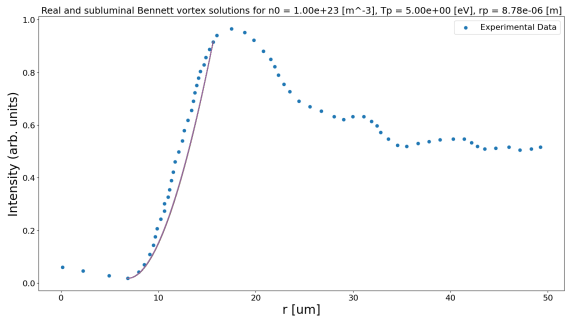


FIG. 3. Shear-flow stabilized solutions for the shear layer in a laser-driven shock of a water microjet using the second closest edge point as obtained from the threshold. The average R^2 of the analytic solutions here is 0.88.

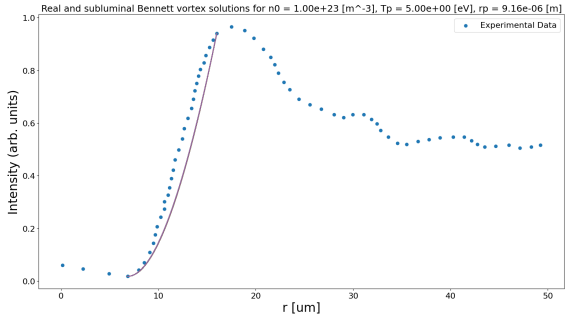


FIG. 4. Shear-flow stabilized solutions for the shear layer in a laser-driven shock of a water microjet using the third closest edge point as obtained from the threshold. The average R^2 of the analytic solutions here is 0.85.

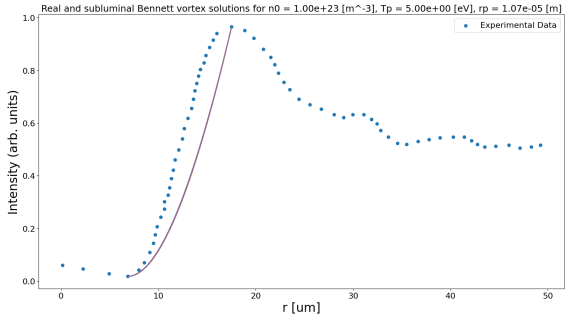


FIG. 5. Shear-flow stabilized solutions for the shear layer in a laser-driven shock of a water microjet using the point of maximal intensity as the edge point. The average R^2 of the analytic solutions here is 0.545. As the edge point moves away from the plateau, and into the shear layer, the fidelity of the analytic solution fades.

TABLE I. Statistical summary for the analytic equilibrium solutions created at each edge point over $NV = 625$ velocity sweeps.

Edge index	Mean R^2	Median R^2	Std. dev.
0	0.903	0.903	0.002
1	0.882	0.882	0.002
2	0.848	0.848	0.005
3	0.544	0.544	0.006

Discussion. It is apparent from Table (I) that there is a sharp decline in the accuracy of the analytic solutions when the edge point is located outside of the shear layer. This occurs due to the lack of saturation in the flow profile here, i.e., it does not fully develop (rise) to the edge state. This lack of development is suggested here to occur because the base point in this modelling was chosen sufficiently close to the shear layer to not leave enough room for the edge-rise to occur even though the boundary conditions remain satisfied.

Nevertheless, it is remarkable that a simplified, 1D, analytic ideal model for a shear-flow stabilized Z-pinch provides such a robust method for calculating accurate solutions to the shear layer of this highly complex physics using experimental data. The complexity of the physics is particularly noteworthy here as it is unknown the number of species present in the plasma, and additionally the problem itself is a multi-phase one as there is a layer of water vapor located around the laser-driven shock event.

These results motivate the study of higher-order effects in this simplified shear-flow stabilized model. They also motivate further investigation into applications for this. One in particular is the research and development of propulsion technology based on the excitation of a fusion plasma pulse in this equilibrium.

Conclusion. A simple 1D, magnetohydrodynamic shear-flow stabilized Z-pinch model was solved using experimental data, and hundreds of accurate solutions were produced, to the shear layer of a laser-driven shock in a water microjet when sweeping over the experimental uncertainty in species velocity. These solutions dip sharply in accuracy as the edge point leaves the shear layer of the system due to the proximity of the base point chosen in the modelling.

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