# The GPT-5.5 Vortex: Another Shear-Flow Stabilized Z-pinch Equilibrium?

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GPT-5.5 was asked to generate another shear-flow stabilized Z-pinch equilibrium. Its choice was to take the cubic, pureflow Bennett-Shumlak-Hartman vortex, which is derived by exchanging the Bennett profile from number density to two-fluid drift velocity,

$$u_z^{(BSH)}(r) = u_{z,0}\frac{r^2}{(r + C_{B,T})^2} \tag{1}$$

and remove the cross-term from the denominator, yielding,

$$u_z^{(GPT)}(r) = u_{z,0}\frac{r^2}{r^2 + C_{B,T}^2} \tag{2}$$

The shear of which is, suppressing the superscript for brevity,

$$\begin{aligned}
u_z' &= u_{z,0}\left(r^2(r^2 + C_{B,T}^2)^{-1}\right)' &(3)\\
&= u_{z,0}\left(2r(r^2 + C_{B,T}^2)^{-1} - 2r^3(r^2 + C_{B,T}^2)^{-2}\right) &(4)\\
&= u_{z,0}\left(\frac{2r(r^2 + C_{B,T}^2)}{(r^2 + C_{B,T}^2)^2} - \frac{2r^3}{(r^2 + C_{B,T}^2)^2}\right) &(5)\\
&= u_{z,0}\left(\frac{2rC_{B,T}^2}{(r^2 + C_{B,T}^2)^2}\right) &(6)
\end{aligned}$$

Then, we can evaluate the Shumlak-Hartman criterion, recalling that this is valid to do with the above profile so long as it represents the MHD flow,

$$\vec{u} = \frac{m_i\vec{u}_i + m_e\vec{u}_e}{m_i + m_e} \tag{7}$$

which is valid, we can recall, when either the electrons are relativistic or stationary. In the relativistic electron case we can use the plasma current density supported by this flow to obtain a self-similar solution for the relativistic electron flow. In the stationary electron case, then we can treat the plasma current density as entirely supported by the ions instead. Both situations give valid reductions to MHD starting from the collisional two-fluid picture.

Therefore, we can evaluate the magnetic field using Ampere's Law,

$$B_\theta(r) = \frac{\mu_0}{r} \int r' J_z(r') dr' \tag{8}$$

which becomes,

$$B_\theta(r) = \frac{\mu_0}{r} \int_0^r r' \frac{r'^2}{(r^2 + C_{B,T}^2)} dr' \tag{9}$$

The integrand is amenable to u-substitution with $u = r'^2 + C_{B,T}^2$,

$$\begin{aligned}
B_\theta(r) &= \frac{en_0 u_{z,0}\mu_0}{r} \int_0^r \frac{r'^3}{r'^2 + C_{B,T}^2} dr' && (10)\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \int_{C_{B,T}^2}^{r^2+C_{B,T}^2} \frac{u - C_{B,T}^2}{u} du && (11)\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \int_{C_{B,T}^2}^{r^2+C_{B,T}^2} 1 - \frac{C_{B,T}^2}{u} du && (12)\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \int_{C_{B,T}^2}^{r^2+C_{B,T}^2} 1 - \frac{C_{B,T}^2}{u} du && (13)\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \left[ u - C_{B,T}^2 \ln(u) \right]_{C_{B,T}^2}^{r^2+C_{B,T}^2} && (14)\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \left( r^2 + C_{B,T}^2 - C_{B,T}^2 \ln\big(r^2 + C_{B,T}^2\big) - C_{B,T}^2 + C_{B,T}^2 \ln\big(C_{B,T}^2\big) \right) &&\\
&= \frac{en_0 u_{z,0}\mu_0}{2r} \left( r^2 + C_{B,T}^2 \ln\left( \frac{C_{B,T}^2}{r^2 + C_{B,T}^2} \right) \right) && (15)
\end{aligned}$$

Does this satisfy the Shumlak-Hartman criterion for Ideal MHD?

$$\frac{\mathrm{d}u_z}{\mathrm{d}r} > 0 \tag{16}$$

From Equation (2) we have a shear given by Equation (6). This shear is always positive and has a value of zero at the origin so the answer appears to be yes, this does seem to satisfy the Ideal MHD version of the Shumlak-Hartman shear-flow stabilization criterion. However, recall as well that this picture depends on $L \to \infty$, so that it can be disrupted if the magnetic field goes to infinity.

What are the limits of the magnetic field given by Equation (15)? Both at the core, and in the limit of infinite plasma radius? The first limit is the most important one because that determines whether an arbitrarily small vortex of this kind can form in a shear-flow stabilized state.

$$\lim_{r\to 0} B_\theta(r) = \lim_{r\to 0} \frac{A}{2}\left(r + \frac{C_{B,T}^2}{r}\ln\left(\frac{C_{B,T}^2}{r^2 + C_{B,T}^2}\right)\right) \tag{17}$$

which is indeterminate in the second argument, so we can apply L'Hopital's rule. The derivative of this logarithmic term is,

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\ln\left(\frac{C_{B,T}^2}{r^2 + C_{B,T}^2}\right)\right) = \frac{1}{g(r)}g'(r) \tag{18}$$

where,

$$g(r) = \frac{C_{B,T}^2}{r^2 + C_{B,T}^2} \tag{19}$$

and,

$$g'(r) = -\frac{2rC_{B,T}^2}{(r^2 + C_{B,T}^2)^2} \tag{20}$$

obtaining,

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\ln\left(\frac{C_{B,T}^2}{r^2 + C_{B,T}^2}\right)\right) = -\frac{2r}{(r^2 + C_{B,T}^2)} \tag{21}$$

Finally giving,

$$\lim_{r\to 0} B_\theta(r) = \lim_{r\to 0} \frac{A}{2}\left(r - C_{B,T}^2\frac{2r}{(r^2 + C_{B,T}^2)}\right) \tag{22}$$

$$= 0 \tag{23}$$

So that we have confirmed that this GPT-5.5 vortex is also a shear-flow stabilized Z-pinch for an arbitrarily small space!

The infinite plasma radius limit should also be considered. In the infinite plasma radius limit, the logarithmic term,

$$\lim_{r\to\infty} \frac{1}{r}\ln\left(\frac{C_{B,T}^2}{r^2 + C_{B,T}^2}\right) = -\frac{\infty}{\infty} \tag{24}$$

is also amenable to L'Hopitalizing. However, the linear factor is not, and it is not indeterminate so it cannot be L'Hopitalized away. Therefore, we can conclude that this profile is not a shear-flow stabilized Z-pinch for an arbitrarily large plasma radius.

The last thing we must determine is what the plasma pressure gradient is. If the plasma pressure does not evaluate, then we are not able to call this an MHD equilibrium. However, it does as can be verified with Wolfram Mathematica,

$$p(r) = p_0 - \frac{A_p}{4}\left(2r^2 + C^2\left(4\ln(C) - 2\ln\big(C^2 + r^2\big) - \ln^2(1 + \frac{r^2}{C^2})\right)\right) \tag{25}$$

where,

$$p_0 = \int_0^{r_p} J_z B_\theta dr \tag{26}$$

$$= \frac{A_p}{4}\left(2r_p^2 + C^2\left(4\ln(C) - 2\ln\bigl(C^2 + r_p^2\bigr) - \ln^2(1 + \frac{r_p^2}{C^2})\right)\right) \tag{27}$$

and we have taken $C_{B,T} \to C$ for brevity.

## Conclusion

GPT-5.5 was asked to come up with a shear-flow stabilized Z-pinch equilibrium. It chose to modify the profile of the cubic, pureflow Bennett-Shumlak-Hartman (BSH) vortex, Equation (1) which is a shear-flow stabilized Ideal MHD equilibrium that requires either relativistic electrons, or stationary electrons. This BSH vortex comes from exchanging the Bennett profile between plasma number density and two-fluid flow velocity. Then, the MHD flow underlying the Shumlak-Hartman criterion can be linked between via either relativistic electrons, or stationary ones.

The modification that it made was to remove the quadratic cross-term from the denominator, obtaining Equation (2). From studying the Shumlak-Hartman criterion it can be seen that this profile is indeed also a shear-flow-stabilized Z-pinch for an arbitrarily small space.