# On the Relativistic Collisional Ordering of a Shear-Flow Stabilized Cubic, Pureflow Bennett-Shumlak-Hartman Vortex

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This brief note shows that the electron fluid momentum reduces to the Ideal Ohm's Law when the ordering $\omega_{pe} << \bar{\nu}_{ei}$ holds for the Braginskii Coulomb collision frequency and *ad hoc* relativistic modifications are made to the equations.

The electron momentum equation reads,

$$m_e n_e \left(\frac{\partial}{\partial t} + \vec{u}_e \cdot \nabla\right)\vec{u}_e = -en_e(\vec{E} + \vec{u}_e \times \vec{B}) - \nabla p_e - m_e n_e \bar{\nu}_{ei}(\vec{u}_e - \vec{u}_i) \tag{1}$$

where the equilibrium-averaged Braginskii electron-ion collision frequency,

$$\bar{\nu}_{ei} = (2.9 * 10^{-6})\frac{n_e \ln(\Lambda)}{T_e^{3/2}} \tag{2}$$

The momentum equation can be rewritten by acknowledging that the electron plasma frequency,

$$\omega_{pe}^2 = \frac{e^2 n_e}{m_e \varepsilon_0} \tag{3}$$

remains invariant under *ad hoc* relativistic modification,

$$n_e = \gamma_e n_{e,0} \tag{4}$$

$$m_e = \gamma_e m_{e,0} \tag{5}$$

$$\therefore \omega_{pe}^2 = \frac{e^2 n_e}{m_e \varepsilon_0} = \frac{e^2 n_{e,0}}{m_{e,0}\varepsilon_0} \tag{6}$$

whereas the Braginskii Coulomb collision frequency does not,

$$\bar{\nu}_{ei} = \gamma_e \bar{\nu}_{ei}^{NR} \tag{7}$$

The above is assuming that the relativistic modifications to the Coulomb logarithm are negligible which is said to be justified here on the basis of the logarithm and extra radical acting upon them.

Making these modifications to the electron momentum equation we have,

$$\gamma_e^2 \frac{\varepsilon_0 m_{e,0}^2}{e^2}\omega_{pe}^2\left(\frac{\partial}{\partial t} + \vec{u}_e \cdot \nabla\right)\vec{u}_e = -e\gamma_e n_{e,0}(\vec{E} + \vec{u}_e \times \vec{B}) - \nabla p_e - \gamma_e^2 \frac{m_{e,0}^2 \varepsilon_0}{e^2}\omega_{pe}^2 \bar{\nu}_{ei}(\vec{u}_e - \vec{u}_i) \tag{8}$$

Dividing through by $\gamma_e^2$ we have,

$$\frac{\varepsilon_0 m_{e,0}^2}{e^2}\omega_{pe}^2\left(\frac{\partial}{\partial t} + \vec{u}_e \cdot \nabla\right)\vec{u}_e = \frac{-en_{e,0}}{\gamma_e}(\vec{E} + \vec{u}_e \times \vec{B}) - \frac{1}{\gamma_e^2}\nabla p_e - \frac{m_{e,0}^2 \varepsilon_0}{e^2}\omega_{pe}^2 \bar{\nu}_{ei}(\vec{u}_e - \vec{u}_i) \tag{9}$$

if the ordering holds,

$$\omega_{pe} << \bar{\nu}_{ei} \tag{10}$$

which it will when the electrons are suitably relativistic then we have,

$$\frac{\varepsilon_0 m_{e,0}^2}{e^2}\frac{\omega_{pe}^2}{\bar{\nu}_{ei}^3}\left(\frac{\partial}{\partial t} + \vec{u}_e \cdot \nabla\right)\vec{u}_e = -\frac{en_{e,0}}{\bar{\nu}_{ei}^3 \gamma_e}(\vec{E} + \vec{u}_e \times \vec{B}) - \frac{1}{\bar{\nu}_{ei}^3 \gamma_e^2}\nabla p_e - \frac{m_{e,0}^2 \varepsilon_0}{e^2}\frac{\omega_{pe}^2}{\bar{\nu}_{ei}^2}(\vec{u}_e - \vec{u}_i) \tag{11}$$

The ordering,

$$\frac{\omega_{pe}^2}{\bar{\nu}_{ei}^2} << 1 \tag{12}$$

is taken here to render the collisional term asymptotically negligible, so that,

$$\frac{\varepsilon_0 m_{e,0}^2}{e^2}\frac{\omega_{pe}^2}{\bar{\nu}_{ei}^3}\left(\frac{\partial}{\partial t} + \vec{u}_e \cdot \nabla\right)\vec{u}_e = -\frac{en_{e,0}}{\gamma_e \bar{\nu}_{ei}^3}(\vec{E} + \vec{u}_e \times \vec{B}) - \frac{1}{\gamma_e^2 \bar{\nu}_{ei}^3}\nabla p_e \tag{13}$$

Then, we can focus on the momentum terms which are all coupled to this inverse squared collisionality term. Along with the Z-pinch ansatz for the electrons,

$$\vec{u}_e = u_e(r)\hat{z} \tag{14}$$

the inertial forces will disappear as well. This leaves us with,

$$\frac{1}{\bar{\nu}_{ei}^3}\left(-\frac{en_{e,0}}{\gamma_e}(\vec{E} + \vec{u}_e \times \vec{B}) - \frac{1}{\gamma_e^2}\nabla p_e\right) = 0 \tag{15}$$

Multiplying again by $\gamma_e^3$ on both sides we have,

$$\frac{\gamma_e^3}{\bar{\nu}_{ei}^3}\left(-\frac{en_{e,0}}{\gamma_e}(\vec{E} + \vec{u}_e \times \vec{B}) - \frac{1}{\gamma_e^2}\nabla p_e\right) = 0 \tag{16}$$

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provided this factor is not infinite, e.g., when $u_e = c$.

The above then becomes,

$$\frac{1}{(\bar{\nu}_{ei}^{NR})^3}\left(-\frac{en_{e,0}}{\gamma_e}(\vec{E}+\vec{u}_e\times\vec{B})-\frac{1}{\gamma_e^2}\nabla p_e\right)=0 \qquad (17)$$

Since $\bar{\nu}_{ei}^{NR}$ is finite the prefactor is non-zero, as otherwise would require $T_e \to \infty$ which would cause the shear localization to collapse, and therefore the momentum balance inside the parentheses must vanish.

Therefore we require,

$$-en_0(\vec{E}+\vec{u}_e\times\vec{B})=\frac{1}{\gamma_e}\nabla p_e \qquad (18)$$

Then, for relativistic electrons the plasma pressure gradient term can be neglected in comparison to the Lorentz force on the electrons, leaving,

$$-en_0(\vec{E}+\vec{u}_e\times\vec{B})\simeq 0 \qquad (19)$$

which requires,

$$\vec{E}=-\vec{u}_e\times\vec{B} \qquad (20)$$

These relativistic modifications are *ad hoc*, and do not replace a fully covariant derivation which is outside the scope of this note. Plasma pressure gradients large enough to overcome the Lorentz factor can also interfere with the above, but the electrons can always be chosen to be sufficiently relativistic that these gradients are outpaced unless $u_e = c$ is required.